Lecture 10

DYNAMIC OPTIMIZATION

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10.1. Graphs, definitions, representations

The term graph was introduced in lecture 9. Further down mostly oriented graphs are examined.

Let's consider a given graph with an ordered pair (V, E) where V is the set of nodes and E is the set of arcs of the graph. Arcs are marked with arrows leading from one node to the other. The arc (P,Q) from node P to node Q is called outgoing for P and incoming for Q.

Definition 1. The starting node is called a node with no incoming arcs, a node without outgoing nodes is called end node of the graph.

Definition 2. A node without any incoming or outgoing arcs is called isolated.

Definition 3. The path in a graph is the ordered set of arcs for which the outgoing node of every arc is incoming for the next.

Definition 4. The contour (cycle) of the graph is a path with coinciding start and end nodes.

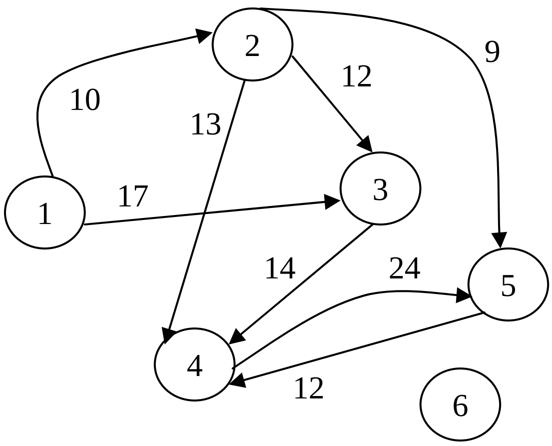
Definition 5. A network is an oriented graph without contours, which has as single starting node and a single ending node.

If every node is juxtaposed to a number which $\in(-\infty,\infty)$ we can say that lengths or weights of the graph are given. Length should be considered in its general meaning. When considered specifically it can be time, profit, expenditure, income, physical length etc.

Example 1. An oriented graph with lengths (weights) of arcs.

Graphs can be represented in different ways. We can demonstrate this for the graph in fig. 1.

A) Presenting through an identity list (for records). It is used for oriented or unoriented graphs. Examples of this type of description are given in fig. 2 and fig. 3. The arrows are pointers towards the neighbors of the corresponding node which is connected to them via arcs. A crossed-out cell is a sign for the end of the corresponding list: nil.



Path: 1, 2, 3, 4
Path: 3, 4, 5, 4
Path: 1, 2, 4, 5,4
Contour: 4, 5, 4

Network: No
Subnetwork: 1, 2, 3, 4 when the rest of the nodes have been removed

Isolated node: 6

Fig. 1. Oriented weight graph.

- Incidence list of an oriented graph

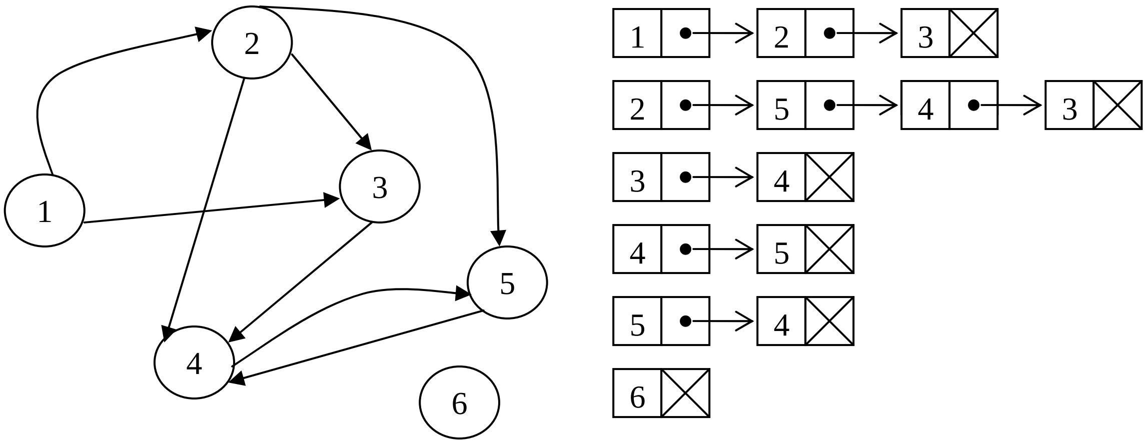


Fig. 2. Oriented graph and records of incidence lists for its nodes.

- incidence lists for an unoriented graph

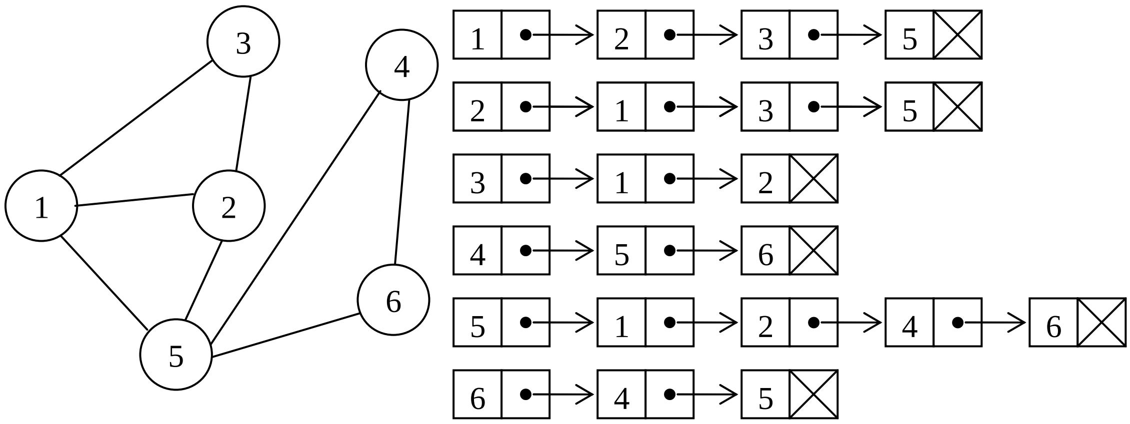


Fig. 3. Unoriented graph and records of incidence lists for its nodes.

B) Representing graphs with an array of vectors

For every arc of the graph a two-dimensional or three-dimensional vector is recorded, containing the numbers of the nodes and their length, which is known, i.e. (P,Q) or (P,Q,d). For example for the graph from fig.2 an array of the following elements can be compiled:

1 2
1 3
2 5
2 4
2 3
3 4
4 5
5 4
6 0

C) Representing a graph by an incidence matrix.

Matrix A is compiled containing elements:

$A_{ij} = \begin{cases} \infty \\ t_{ij} \end{cases}$	∞	- if node i isn't directly connected to node j
	t_{ij}	- a number indicating the length of the arc between node i and node j

For the example from fig. 1 the incidence matrix is

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} \infty & 10 & 17 & \infty & \infty & \infty \\ \infty & \infty & 12 & 13 & 9 & \infty \\ \infty & \infty & \infty & 14 & \infty & \infty \\ \infty & \infty & \infty & \infty & 24 & \infty \\ \infty & \infty & \infty & 12 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \end{bmatrix}.$$

D) Representing a graph by distance matrix. Matrix R is compiled containing elements like those of incidence matrix A, save for the distances of the main diagonal, where the distance is always zero:

$$R = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 0 & 10 & 17 & \infty & \infty & \infty \\ \infty & 0 & 12 & 13 & 9 & \infty \\ \infty & \infty & 0 & 14 & \infty & \infty \\ \infty & \infty & \infty & 0 & 24 & \infty \\ \infty & \infty & \infty & 12 & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix}.$$